II. *A Letter to Dr.* Mead, *Coll. Med.* Lond. *& Soc. Reg. S. concerning an Experiment, whereby it has been attempted to ſhew the falſity of the common Opinion, in relation to the force of Bodies in Motion. By* Henry Pemberton, *M. D.* R. S. S.

SIR,

PEruſing the Learned *Polenus*'s Tract *de Caſtellis*, you were pleaſed to ſend me; I have found in it ſeveral curious Experiments, among which I reckon that of letting Globes of equal Magnitude, but of different weights, fall upon a yielding Subſtance, as Tallow, Wax, Clay or the like, from heights reciprocally proportional to the weights of the Globes. This Experiment engaged in particular my Attention, as it is brought with deſign to overturn one of the firſt Principles eſtabliſh'd in Natural Philoſophy. And the Knowledge I have of your great Eſteem for that part of Science, emboldens me to trouble you with my Thoughts upon this Experiment; for I cannot by any means admit of the Deduction that is drawn from thence, that becauſe the Globes make in this Experiment equal Impreſſions in the yielding Subſtance, therefore they ſtrike upon it with equal force: whereby it is attempted to prove the Aſſertion of Mr. *Leibnitz*, that the force of the ſame Body in deſcending is proportional to the height from whence it falls; or, in all Motion, proportional to the Square of the Velocity, and not proportional to the Velocity it ſelf, as is commonly thought. On the contrary, I think this very Experi-

ment proves the great unreaſonableneſs of Mr, *Leibnitz*'s Notion.

I am ſurpriz'd, that ſo careful a Writer as *Polenus* appears to be, from the accuracy wherewith he delivers his Experiments, ſhould not rather ſuſpect his reaſoning in an intricate Caſe, than thus contradict a Principle in Philoſophy, that has been directly prov'd by a multitude of Experiments, in particular by thoſe Sir *Iſaac Newton* recommends for that purpoſe (*a*); and that is moreover abundantly eſtabliſh'd by its exact Agreement with all Obſervations; as being the Principle upon which all appearances, hitherto obſerved in the motion of Bodies, are accounted for by juſt and undeniable Arguments; and we ſhall find on Inquiry, that the preſent Caſe comes alſo under the ſame Rule.

As the uſe of Experiments in Natural Philoſophy is to diſcover the Cauſes of Things, by exhibiting more ſimple Effects of thoſe Cauſes, than occur in the ordinary courſe of Nature; ſo for this end it is neceſſary, that our Argumentation upon Experiments be perfectly juſt, otherwiſe they may lead us into Errors. The firſt thing neceſſary for making right Deductions from an Experiment, is to determine the proper uſe thereof; which I think in this before us is not rightly underſtood. Certainly this Experiment of *Polenus* is much more fit to inform us of the Law, by which theſe yielding Subſtances reſiſt the motion of Bodies ſtriking upon them, than to ſhew the forces, with which Bodies ſtrike; for whatever thoſe forces be, the Effects muſt be very different, according to the Difference there may be in the Rule obſerved by ſuch reſiſtance.

(*a*) Princip. Philoſ. Nat. *p.* 19.

Now this Experiment ſhews, that if two Globes in Motion bear againſt equal Portions of the yielding Subſtance, the Oppoſition, that Subſtance makes to the Motion of the Globes, will be the ſame in both, however different the Velocities be, with which they move. This I ſhall demonſtrate as follows.

Let *A* and *B* be two Globes, equal in Magnitude, but of different Weights, which are equally immerſed into a yielding Subſtance. Suppoſe the Velocities, with which they move in their preſent Situation, to be reciprocally in the Subduplicate *ratio* of the Weights of the Globes; that is, let the *ratio* of the Weight of the Globe *A* to the Weight of the Globe *B*, be Duplicate of the *ratio* of the Velocity of the Globe *B*, to the Velocity of the Globe *A*. Since therefore the *ratio* of the quantity of Motion in the Globe *A*, or of the force with which it moves, to the quantity of Motion in the Globe *B*, or to the force with which that Globe moves, is compounded of the *ratio* of the Weight of the Globe *A*, to the Weight of the Globe *B*, and of the *ratio* of the Velocity of the Globe *A*, to the Velocity of the other Globe *B*, the force, with which the Globe *A* moves, is to the force, with which the Globe *B* moves, as the Velocity of this Globe *B*, to the Velocity of the other Globe *A*. But if the ſame Oppoſition be made to the Motion of the Globes, when they bear upon equal Portions of the yielding Subſtance, the Effect of that Oppoſition, while the Globes enter farther into the Subſtance by equal Spaces, will be proportional to the time, in which the Globes are moving thoſe Spaces, or in which the Oppoſition is made, if we conſider thoſe Spaces while naſcent or in their firſt Origine; the Effect therefore of this Oppoſition will be reciprocally proportional to the Velocity of each Globe: namely, the momentaneous loſs of force in the Globe

A will be to the momentaneous loſs of force in the Globe *B*, as the Velocity of the Globe *B*, to the Velocity of the Globe *A*; and the whole force of the Globe *A* has been found to bear the ſame *ratio* to the whole force of the Globe *B*; conſequently theſe Globes, while they penetrate equal Spaces into the Subſtance, loſe parts of their force, which bear the ſame proportion to the whole: and therefore, if their Velocities be at any time reciprocally in the Subduplicate *ratio* of their Weights, ſo that the forces or degrees of Motion, with which they move, be reciprocally proportional to their Velocities, the forces, with which they preſs into the yielding Subſtance, at equal Indentures made in the Subſtance, will continue in the ſame Proportion; and therefore upon the Theory of Reſiſtance here ſuppoſed, when the whole Force and Motion of both theſe Globes is entirely loſt, they will be plunged into the Subſtance at equal Depths.

Now whereas in the Experiment of *Polenus*, the Globes, falling from Heights reciprocally proportional to their Weights, ſtrike upon the yielding Subſtance with Velocities reciprocally in the Subduplicate proportion of their Weights, and the Effect is in all Caſes found to be, what is here deduced from the Theory of Reſiſtance, I have propoſed; it is a ſufficient Confirmation of the Truth of this Theory.

Only here, Sir, I ought to obſerve to you, That I have ſuppoſed the Globes to be ſtopt by the whole Reſiſtance of the Subſtance, they move againſt; although in ſtrictneſs they are ſtopt only by the exceſs of that Reſiſtance above the Action of Gravity upon them. But I have neglected the Conſideration of the Action of Gravity, that being but ſmall in Proportion to the Reſiſtance, as will appear from the Globes being much more ſpeedily ſtopt by this Reſiſtance, than they

they would be by the Action of Gravity, if its force were applied upwards; for by that force alone, the Globes would not be ſtopt, till they had meaſured Spaces equal to the heights above the reſiſting Subſtance, from whence they fell; which heights bear a great Proportion to the depths, the Globes in this Experiment are immerſed into the yielding Subſtance, as I have found upon trial.

Thus, if I miſtake not, may be removed the Difficulty attending this Experiment. But as Mr. *Leibnitz*'s Opinion is deduced from it by Means of this Axiom, that Effects are proportional to their Cauſes; ſo that here the Effects being thought the ſame, the Cauſes are concluded to be ſo likewiſe; it will not be amiſs to mention in this place an Experiment, where this Axiom may be more juſtly applied, than it can be in our preſent caſe, from which Experiment the received Opinion may be proved. This Experiment is mentioned by *Polenus* from *Merſennus*, though tried by him ſomewhat rudely; but has been often ſince made in the following manner. To one end of the Beam of a Ballance is hung a Weight, and from a proper height is let fall upon the other extremity of the Beam another Weight, which by ſtriking thereon ſhall raiſe the end, to which the Weight is ſuſpended, to ſuch a height, as is juſt ſufficient to ſet free a certain Spring. If then a different Weight be hung in the room of the former; the height, from whence the falling Weight muſt deſcend, in order to raiſe the Extremity of the Ballance, to which this other Weight is ſuſpended, to the ſame height as before, that is, high enough to ſet the forementioned Spring at liberty, is found to be ſuch, that the Velocity, with which the falling Weight ſtrikes upon the Ballance, in this latter Caſe, will be to its former Velocity, as the latter Weight

to

to the former, (*a*) except only that, agreeably to what *Merſennus* himſelf remarked, when the Weight deſcends from great heights, an Altitude ſomewhat greater than this Rule implies is required to raiſe the other Weight as much as is deſired. But whether the bending of the Arm of the Ballance, when acted upon with a great force, or whether any increaſe of Friction in this Caſe, occaſion the Irregularity here mentioned, we need not ſtrictly enquire; for this Irregularity is yet leſs reconcileable with the new Opinion, than the regular Effects of the Experiment. Hence therefore we may ſee, that the very Method of reaſoning, which being applied erroneouſly, is ſuppoſed to prove Mr. *Leibnitz*'s Sentiment concerning the force of Bodies in Motion, will, when juſtly uſed, confirm the other Opinion in relation to that Matter.

But as I have aſſerted in the beginning of this Letter, that the very Experiment of *Polenus* is not only reconcileable to the common Doctrine of Motion, as I have now demonſtrated; but even that it does it ſelf make manifeſt the great unreaſonableneſs, if not the abſolute Abſurdity, of Mr. *Leibnitz*'s Opinion; it remains that I briefly make proof of this.

If two Globes *A* and *B*, of equal Magnitude but of different Weights, ſtriking on a yielding Subſtance with equal force, in every Caſe loſe all their Motion at equal Depths, it is neceſſary that at all times, during their Motion, they loſe equal Degrees of force, when they bear upon equal Portions of the Subſtance, in entring equal Spaces into the Subſtance. This will be eaſily ſeen from what has before been ſaid. Now whereas Mr. *Leibnitz* ſuppoſes the power of Gravity to

(*a*) *Vid.* 's Graveſande Phyſic. Elem. Tom. I. p. 39.

give

give to the ſame falling Body Degrees of force proportional to the height from whence it falls; according to his Opinion, by the power of Gravity, equal Degrees of force are added in the deſcent of the ſame Body through equal Spaces; and in different Bodies deſcending through equal Spaces, the Degrees of force added will be as the quantity of Matter, or as the weight of each Body. Therefore while the Globes *A* and *B* penetrate equal naſcent Spaces into the yielding Subſtance, by the Action of Gravity, were not that Action overcome by the Reſiſtance of that Subſtance, additional Degrees of force would be communicated in ſuch proportion, that the force added to the Globe *A*, would be to the force added to the Globe *B*, as the weight of the Globe *A*, to the weight of the Globe *B*, or in the Duplicate *ratio* of the Velocity of the Globe *B*, to the Velocity of the Globe *A*. But ſince the Globes loſe the ſame Degrees of force in entring equal Naſcent Spaces into the yielding Subſtance, the Effect of the Oppoſition made by this Subſtance to the Motion of the Globes, during the time of their paſſing through ſuch Naſcent Spaces, will be both the taking from them that ſame Degree of force, and moreover the additional force, which would otherwiſe have been given them by their own Gravity. But farther, the Oppoſition made to the motion of the Globe *A*, to the Oppoſition made to the motion of the Globe *B*, will be in the *ratio* compounded of the *ratio* of the Effect of the Oppoſition, the Subſtance makes to the motion of the Globe *A*, to the Effect of the Oppoſition, the Subſtance makes to the motion of the Globe *B*, and of the *ratio* of the Time, in which the Oppoſition is made againſt the latter Globe, to the Time in which it is made againſt the former; which latter *ratio* is the

ſame

ſame with the *ratio* of the Velocity of the Globe *A*, to the Velocity of the Globe *B*. But ſince it is ſhewn, that the Effect of the Oppoſition made by the yielding Subſtance to theſe Globes is two-fold, and that one part of the Effect of the Oppoſition made to the motion of the Globe *A*, is equal to one part of the Effect of the Oppoſition made to the motion of the Globe *B*; and that another part of the Effect of the Oppoſition made to the motion of the Globe *A*, to another part of the Effect of the Oppoſition made to the motion of the Globe *B*, is in the Duplicate *ratio* of the Velocity of the Globe *B*, to the Velocity of the Globe *A*. One part of the Oppoſition it ſelf made to the motion of the Globe *A*, will be to one part of the Oppoſition againſt the motion of the Globe *B*, as the Velocity of the Globe *A*, to the Velocity of the Globe *B*, and another part of the Oppoſition to the motion of the Globe *A*, to another part of the Oppoſition to the motion of the Globe *B*, will be as the Velocity of the Globe *B*, to the Velocity of the Globe *A*. So that when the Globes bear upon equal Portions of the yielding Subſtance, the Oppoſition to their Motion will be in part as the Velocity of the Globes, and in part reciprocally as their Velocity. Hence, becauſe the reſiſting Subſtance is of an uniform Texture, the Oppoſition to the Motion of either of the Globes in its preſent Situation, and when moving with its preſent Velocity, will be to the Oppoſition it would meet with in the ſame Situation, if it moved with any other Velocity, partly as the preſent Velocity to that other Velocity, and partly as that other Velocity to the preſent. But by that part of the Oppoſition made againſt the motion of the Globe, which is directly as the Velocity, the Globe can never be wholly ſtopt; for upon the ſtopping of the Globe, that part of the Oppoſi-

tion

tion to its Motion will likewiſe totally ceaſe, and conſequently the Globe's Weight will carry it further down, unleſs the other part of the Oppoſition againſt its Motion prevent it. But I ſay again, neither can this latter part of the Oppoſition made to its Motion be ever great enough to ſtop the Globe; for the Degree of this Oppoſition being reciprocally as the Velocity of the Globe, when the motion of the Globe is wholly taken away, it will become infinitely greater, than at any time, while the Globe is in Motion; ſo that when the Globe ſhould be ſtopt by this part of the Oppoſition made to its Motion, the Oppoſition to the Globe's Motion will become infinitely great; inſomuch, that no Degree of force whatever could be able to impel the Globe further into the Subſtance, but this can never come to paſs. Beſides, it is not neceſſary to apply any ſuch refined Argument againſt this part of the Reſiſtance; it would be alone ſufficient to conſider, how unreaſonable a Suppoſition it is, that a Reſiſtance ſhould increaſe, when the Velocity of the reſiſted Body decreaſes.

Thus may this Experiment be made uſe of to invalidate that very Opinion it is brought to ſupport. But another uſe may likewiſe be made thereof: For it will ſerve to illuſtrate what the great Sir *Iſaac Newton* has more than once hinted, that the Reſiſtance of Fluids, which ariſes from the Tenacity of their Parts, decreaſes in a leſs Proportion than the Velocity of the reſiſted Body decreaſes (*a*); for as this Reſiſtance bears a great Analogy to the Reſiſtance of the yielding Subſtances we have been here treating of, ſo we have found, that the Reſiſtance of theſe Subſtances does

(*a*) *Vid.* Philoſ. Nat. Princip. Math. Prop. 52. lib. 2. in Schol. Opticks. Qu. 28. p. 339, 340.

not much depend upon the Velocity of the Body, againſt which the Reſiſtance is applied.

And thus, Sir, we may ſee how all Experience conſpires in confirming and ſetting forth that ſtupendious force of reaſoning, which has enabled our great Philoſopher moſt ſurprizingly to ſearch out, and diſtinguiſh the Springs of Natural Operations; a Work infinitely more difficult to accompliſh than even the great Improvements he has made in pure Mathematicks, which were previouſly neceſſary in order to his ſucceeding in his Searches after the Knowledge of Nature; for in this laſt Purſuit he has given Proof, not only of a more unbounded Invention, than is required in the ſubtileſt Geometrical Speculations; but has alſo there diſcovered the greateſt Diſcernment, and moſt conſummate Judgment; ſince in his Philoſophical Writings, he has never been once impoſed on by an Hypotheſis, nor by any other of the various Fallacies, which my Lord *Bacon* in his *Novum Organon*, has reckon'd up as the Cauſes, that had hindred the Improvement of the true Philoſophy.

But here I ſhall put an end to this long Letter; for the Freedom of which, I know, I need make no Apology to you, Sir, of whoſe great Candour I have for ſome Years paſt been a conſtant Witneſs; and as I have frequently admired, how amidſt the vaſt Employments of your Profeſſion, you ſhould find time to purſue with ſo great Succeſs ſuch various ſorts of Learning; ſo I have as often been delighted to obſerve with what Benignity you receive thoſe, who have made any the leaſt part of uſeful Knowledge their Study.

I am, Sir, &c.

H. P.

POST-SCRIPT.

ABout a Week after I had ſent you the Letter, containing my Obſervations on *Polenus*'s Experiment, I had the good Fortune to hear an excellent and learned Friend of yours, to whom you had been pleaſed to ſhew my Letter, give a very curious and weighty Argument to confirm Sir *Iſaac Newton*'s Sentiment in relation to the Reſiſtance of Fluids, which I had deduced from the above mentioned Experiment; and as this very much pleaſed me, I ſhall here endeavour to ſend you an Account of it in the following manner:

Suppoſe pieces of fine Silk, or the like thin Subſtance, extended in Parallel Planes, and fixt at ſmall Diſtances from each other. Suppoſe then a Globe to ſtrike perpendicularly againſt the middle of the outermoſt of the Silks, and by breaking through them to loſe part of its Motion. If the pieces of Silk be of equal Strength, the ſame Degree of force will be required to break each of them; but the Time, in which each piece of Silk reſiſts, will be ſo much ſhorter as the Globe is ſwifter; and the loſs of Motion in the Globe conſequent upon its breaking through each Silk, and ſurmounting the Reſiſtance thereof, will be proportional to the Time, in which the Silk oppoſes itſelf to the Globe's Motion; inſomuch that the Globe by the Reſiſtance of any one piece of Silk will loſe ſo much leſs of its Motion as it is ſwifter. But on the other Hand, by how much ſwifter the Globe moves,

ſo many more of the Silks it will break through in a given Space of Time; whence the number of the Silks, which oppoſe themſelves to the Motion of the Globe in a given Time, being reciprocally proportional to the Effect of each Silk upon the Globe, the Reſiſtance made to the Globe by theſe Silks, or the loſs of Motion, the Globe undergoes by them in a given Time, will be always the ſame.

Now if the Tenacity of the Parts of Fluids obſerves the ſame Rule as the Coheſion of the Parts of theſe Silks; namely, That a certain Degree of force is required to ſeparate and diſunite the adhering Particles, the Reſiſtance ariſing from the Tenacity of Fluids muſt obſerve the ſame Rule as the Reſiſtance of the Silks, and therefore in a given Time the loſs of Motion, a Body undergoes in a Fluid by the Tenacity of its Parts, will in all Degrees of Velocity be the ſame; or in fewer Words, that part of the Reſiſtance of Fluids, which ariſes from the Coheſion of their Parts, will be Uniform.
